Karl-Heinz Spatschek

Introduction to Theoretical Plasma Physics

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Chapter 1
Introduction

A plasma is a many body system of a huge number of interacting (charged and neutral) particles. More precisely, we define a plasma as a quasi-neutral gas of charged and neutral particles which exhibits collective behavior. In most cases, we shall consider quasi-neutral gases consisting of charged particles, namely electrons and positively charged ions of one species. Plasma may contain neutral atoms. In this case, the plasma is called partially or incompletely ionized. Otherwise the plasma is said to be completely or fully ionized. In the next sections we shall explain what we understand by quasi-neutral and collective behavior. The term plasma is not limited to the most common electron-ion case. One talks about electron-positron plasmas, quark-gluon plasmas. Semiconductors contain plasma consisting of electrons and holes.

Most of the matter in the Universe exists in the plasma form. It is often said that 99% of the matter in the universe is in the plasma state. But this estimate may not be very accurate. Remember the discussion on dark matter, but certainly the plasma state is dominating in the universe.

Modern plasma physics emerged in 1950's, when the idea of thermonuclear reactor was put forward. It should not be concealed that this activity was started by the H-bombs developed in the U.S. and the U.S.S.R. in 1952 and 1953. Fortunately, modern plasma physics completely dissociated from weapon development. The progression of modern plasma physics can be retraced, e.g., from the following monographs shown under Refs. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

In general plasmas can be characterized by two parameters, namely charged particle density n and temperature T. The density varies over approx. 28 orders of magnitude, from 10^6 to 10^{34} m^{-3}. The kinetic energy $k_B T$, where k_B is the Boltzmann constant, can vary over seven orders from 0.1 to 10^6 eV. The following nomogram Fig. 1.0.1 gives an overview.

The term "plasma" was introduced by Langmuir, Tonks, and their collaborators in the 1920's when they studied processes in electronic lamps filled with ionized gases, i.e. low-pressure discharges. The word "plasma" seems to be a misnomer [14]. The Greek πλάσμα means something modelled or fabricated. However, a

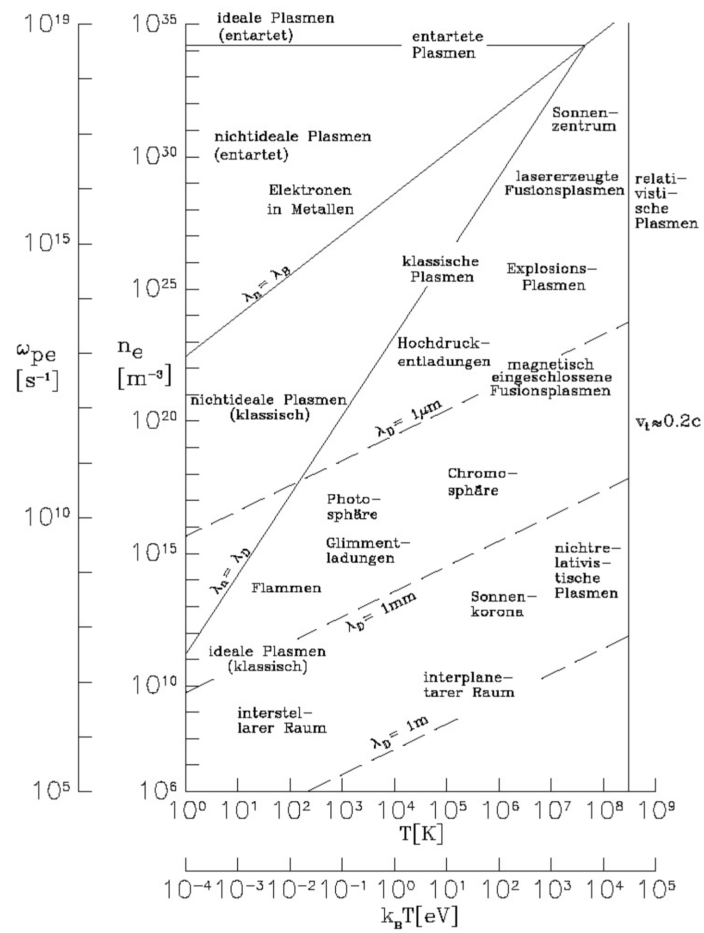

Fig. 1.0.1 Simple view of the occurrence of plasmas in physical parameter space.

plasma does not tend in general to conform to external influences. On the contrary, because of collective behavior, it often behaves as if it had a mind of its own [14].

Plasmas appear in space and astrophysics [15, 16, 17], laser-matter-interaction [18, 19], technology [20], fusion [21, 22, 23, 24, 25], etc. We will discuss aspects of these applications during the course of lectures.

Technical plasmas, magnetic fusion plasmas and laser-generated plasmas are the main applications of plasma physics on earth. Space plasmas, as they appear, e.g., in the magnetosphere of the earth (see Fig. 1.0.2) are very important for our life on earth. A continuous stream of charged particles, mainly electrons and protons,

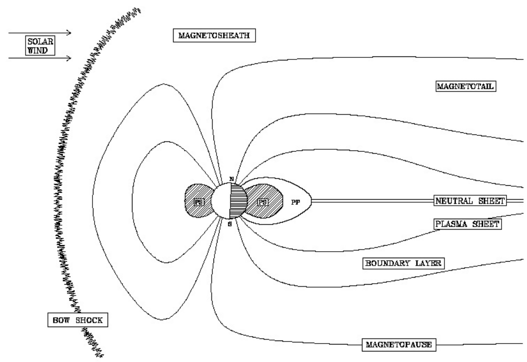

Fig. 1.0.2 Schematic plot of the earth and the magnetosphere

called the solar wind, impinges on the earth's magnetosphere which shields us from this radiation. Typical solar wind parameters are $n = 5 \times 10^6 m^{-3}$, $k_B T_i = 10\,eV$, and $k_B T_e = 50\,eV$. But also temperatures of the order $k_B T = 1\,keV$ may appear. The drift velocity is approx. 300 km/s.

We could present numerous other examples of important plasmas, ranging from stellar interiors and atmospheres which are hot enough to be in the plasma state, over the stars in galaxies, which are not charged but behave like particles in a plasma, to the free electrons and holes in semiconductors which also constitute a plasma. But let us develop the theory step by step.

1.1 Quasi-neutrality and Debye shielding

Negative charge fluctuations $\delta\rho = -e\delta n$ (e is the elementary charge) generate electrostatic potential fluctuations $\delta\phi$ (we use the Gaussian system; see below)

$$\nabla^2 \delta\phi = 4\pi e \delta n \, . \tag{1.1.1}$$

A rough estimate gives

$$\nabla^2 \delta\phi \sim \frac{\delta\phi}{l^2} \, , \tag{1.1.2}$$

where l is the characteristic fluctuation scale. Thus,

$$\delta\phi \approx 4\pi e \delta n l^2 \, . \tag{1.1.3}$$

On the other hand, the characteristic potential energy $-e\delta\phi$ cannot be larger than the mean kinetic energy of particles which we roughly approximate by $k_B T$ (we measure the temperature in Kelvin, k_B is the Boltzmann constant, and we ignore numerical factors). Thus,

$$\frac{\delta n}{n} \lesssim \frac{k_B T}{4\pi n e^2 l^2} . \tag{1.1.4}$$

We recognize that a typical length appears, namely the Debye length (more details will be given within the next chapters)

$$\lambda_D = \sqrt{\frac{k_B T}{4\pi n_{e0} e^2}} , \tag{1.1.5}$$

such that

$$\frac{\delta n}{n} \lesssim \frac{\lambda_D^2}{l^2} . \tag{1.1.6}$$

We will now present arguments that a plasma is quasi-neutral on distances much larger than the Debye radius. If the plasma size is comparable with λ_D, then it is not a "real" plasma, but rather just a heap of charged particles.

The Debye length is the shielding length in a plasma (we do not discuss at the moment which species, electrons or ions, are dominating; see next chapter). Let us again start from the Poisson equation

$$\nabla^2 \phi = 4\pi e(n_e - n_i) . \tag{1.1.7}$$

Assuming Boltzmann distributed electrons and ions (let us assume with the same temperature T which we measure in eV, i.e. $k_B T \to T$), we have

$$n_e = n_{e0} \mathrm{e}^{e\phi/T} \approx n_{e0}(1 + e\phi/T) , \quad n_i = n_{e0} \mathrm{e}^{-e\phi/T} \approx n_{e0}(1 - e\phi/T) . \tag{1.1.8}$$

Assuming spherical symmetry, we obtain

$$\nabla^2 \phi = \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{8\pi e^2 n_{e0}}{T}\phi \tag{1.1.9}$$

It is easy to check that the solution is

$$\Phi = -\frac{e}{4\pi\varepsilon_0 r} \mathrm{e}^{-\sqrt{2}r/\lambda_D} , \quad \lambda_D = \sqrt{\frac{T}{4\pi n_{e0} e^2}} . \tag{1.1.10}$$

Thus the potential in a plasma is exponentially shielded with the Debye length as the shielding distance.

The picture of the Debye shielding is valid only if there are enough particles in the charge cloud. We can compute the number N_D of particles in a Debye sphere,

$$N_D = n\,\frac{4}{3}\pi\lambda_D^3 ; \tag{1.1.11}$$

effective shielding over a Debye length requires

$$N_D \gg 1 \,. \tag{1.1.12}$$

A plasma with characteristic dimension L can be considered quasi-neutral, provided

$$\lambda_D \ll L \,. \tag{1.1.13}$$

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